

Congruent Triangles

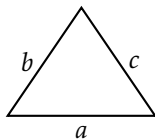
Things you should already know

Fact (Congruence) — Two shapes are **congruent** if they are exactly the same shape and size. Congruent shapes can be reflections or rotations of each other.

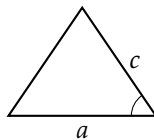
Fact (Angle Sum) — Angles in a triangle sum to 180° .

Fact — Two triangles are **congruent** if any one of the following conditions holds:

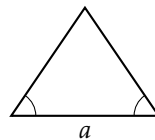
- **SSS** — all three pairs of sides are equal
- **SAS** — two pairs of sides and the **included** angle are equal
- **ASA** (or **AAS**) — two pairs of angles and a corresponding side are equal
- **RHS** — both have a right angle, equal hypotenuses, and one other equal side



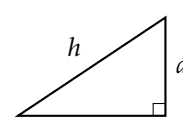
SSS



SAS



ASA



RHS

Example

Why is **SSA** (two sides and a non-included angle) **not** sufficient for congruence?

Example

Why is **AAA** not sufficient for congruence?

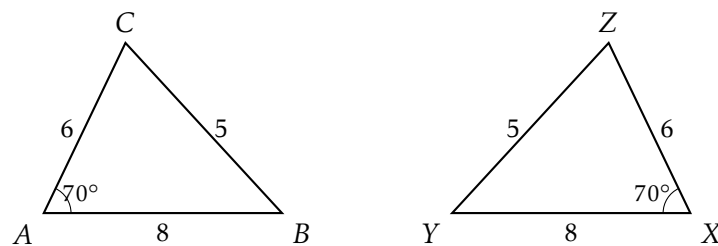
Example

State the congruence condition for each pair:

- (a) $\triangle ABC$ with $AB = 5$, $BC = 7$, $AC = 8$ and $\triangle DEF$ with $DE = 5$, $EF = 7$, $DF = 8$.
- (b) $\triangle PQR$ with $PQ = 6$, $\angle Q = 90^\circ$, $QR = 8$ and $\triangle XYZ$ with $XY = 6$, $\angle Y = 90^\circ$, $YZ = 8$.
- (c) $\triangle ABC$ with $\angle A = 40^\circ$, $AB = 9$, $\angle B = 70^\circ$ and $\triangle DEF$ with $\angle D = 40^\circ$, $DE = 9$, $\angle E = 70^\circ$.

Example

Write corresponding vertices in the correct order.



Similar Triangles

Example

A triangle has angles 40° , 60° , 80° and sides 5 cm, 6 cm, 7 cm. Another triangle has angles 40° , 60° , 80° and sides 10 cm, 12 cm, 14 cm. Are they congruent? What is special about them?

Fact — Two triangles are **similar** if any one of the following conditions holds:

- **AAA** — two (and hence all three) pairs of angles are equal
- All three pairs of sides are **in the same ratio**
- Two pairs of sides are in the same ratio and the included angles are equal

If two triangles are similar, then all corresponding sides are in the same ratio (the **scale factor**).

Finding Unknown Lengths

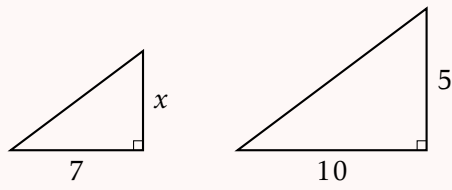
There are two methods:

Method 1 — Between triangles: compare corresponding sides across the two triangles.

Method 2 — Within triangles: compare sides within the same triangle.

Example

The triangles below are similar. Find x .



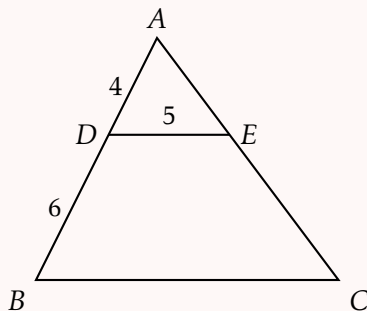
Spotting Similar Triangles

Fact — Look for similar triangles when you see:

- Parallel lines cutting through two sides of a triangle (by alternate/corresponding angles)
- Triangles sharing an angle, with other angles equal
- “Butterfly” configurations (two triangles sharing a vertex)

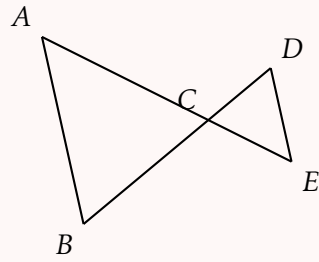
Example

In the diagram, DE is parallel to BC . $AD = 4$ cm, $DB = 6$ cm, $DE = 5$ cm. Find BC .



Example

In the diagram, BCD and ACE are straight lines. $AC = 12$, $BC = 8$, $CD = 6$, $CE = 4$. Show that $\triangle ABC$ is similar to $\triangle DEC$.



Ratios in Similar Figures**Example**

In the diagram, $XY \parallel BC$. $AX : XB = 2 : 3$. If $BC = 15$ cm, find XY .

